Nambu-like odd brackets on supermanifolds

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ABSTRACT: The Grassmann-odd Nambu-like brackets corresponding to an arbitrary Lie superalgebra and realized on the supermanifolds are proposed.

Dedicated to Valentina Erofeyevna Korol' and Nikita Dmitriyevich Soroka.

Contents

1. Introduction

In the paper [1] the Grassmann-odd linear Poisson bracket corresponding to the $SO(3)$ group and built up only of the Grassmann variables has been introduced. It was found for this bracket at once three Grassmann-odd nilpotent Batalin-Vilkovisky type Δ-like differential operators [2, 3] of the first, second and third orders with respect to the Grassmann derivatives. Then [4] the Nambu [5] Grassmann-odd bracket on the Grassmann algebra was constructed with use of the third order Δ-operator.

In [6, 7, 8] the linear Grassmann-odd Poisson bracket corresponding to an arbitrary Lie group and realized solely on the Grassmann variables has been proposed. Again it was found for this bracket three Grassmann-odd nilpotent Δ-like differential operators of the first, second and third orders with respect to the Grassmann derivatives. Later [9] with the help of the third order Δ-operator the Nambu-like Grassmann-odd bracket on the Grassmann algebra was developed.

The purpose of the present paper is to extend this construction onto the arbitrary Lie supergroup realized on the supermanifolds. In particular, the linear Grassmann-odd Poisson bracket corresponding to the arbitrary Lie supergroup is constructed on the supermanifolds. Then for this bracket six Grassmann-odd nilpotent Δ-like differential operators of the first, second and third orders with respect to the derivatives are obtained. At last, for every Lie supergroup by means of two third order Δ-operators two different Nambu-like Grassmann-odd brackets are built up on the supermanifolds.

2. Linear odd Poisson bracket on supermanifolds

The operator Π of the Grassmann parity inversion (see, e.g., [10]) acts on the coordinates z_α of the co-adjoint representation G^* of the Lie superalgebra G in the following way:

$$\Pi z_\alpha = c_\alpha, \qquad p(c_\alpha) = p(z_\alpha) + 1 \pmod 2,$$

where c_α are coordinates in ΠG^* and $p(z_\alpha)$ is a Grassmann parity of the value z_α.

The linear Grassmann-odd Poisson bracket corresponding to the arbitrary Lie supergroup and realized on the supermanifolds has the following form:

$$\{A, B\}_1 = A \overleftarrow{\partial}_{c_\alpha} f_{\alpha\beta}{}^\gamma c_\gamma \overrightarrow{\partial}_{c_\beta} B, \tag{2.1}$$

where $f_{\alpha\beta}{}^\gamma$ are structure constants entering in the permutation relations

$$G_\alpha G_\beta - (-1)^{p(z_\alpha)p(z_\beta)} G_\beta G_\alpha = f_{\alpha\beta}{}^\gamma G_\gamma$$

for the generators G_α of the Lie superalgebra G, $\overleftarrow{\partial}$ and $\overrightarrow{\partial}$ are the right and left derivatives, $\partial_{c_\alpha} \equiv \frac{\partial}{\partial c_\alpha}$ and A, B are functions of c_α. The structure constants $f_{\alpha\beta}{}^\gamma$ have a Grassmann parity

$$p(f_{\alpha\beta}{}^\gamma) = p(z_\alpha) + p(z_\beta) + p(z_\gamma) = 0 \pmod 2,$$

following symmetry properties:

$$f_{\alpha\beta}{}^\gamma = -(-1)^{p(z_\alpha)p(z_\beta)} f_{\beta\alpha}{}^\gamma \tag{2.2}$$

and obey the Jacobi identity

$$\sum_{(\alpha\beta\gamma)} (-1)^{p(z_\alpha)p(z_\gamma)} f_{\alpha\lambda}{}^\mu f_{\beta\gamma}{}^\lambda, \tag{2.3}$$

where the symbol $(\alpha\beta\gamma)$ means a cyclic permutation of the quantities α, β and γ.

The bracket (2.1) possesses at once six Grassmann-odd nilpotent Batalin-Vilkovisky type [2, 3] Δ-like differential operators of the first, second and third orders with respect to the derivatives

$$\Delta_{+1} = \frac{1}{2}(-1)^{p(c^\alpha)} c^\alpha c^\beta f_{\beta\alpha}{}^\gamma \partial_{c^\gamma}, \tag{2.4}$$

$$\tilde{\Delta}_{+1} = \frac{1}{2}(-1)^{p(c^\alpha)} c^\alpha c^\beta f_{\alpha\beta}{}^\gamma \partial_{c^\gamma}, \tag{2.5}$$

$$\Delta_{-1} = \frac{1}{2}(-1)^{p(c_\alpha)} f_{\alpha\beta}{}^\gamma c_\gamma \partial_{c_\beta} \partial_{c_\alpha}, \tag{2.6}$$

$$\tilde{\Delta}_{-1} = \frac{1}{2}(-1)^{p(c_\beta)} f_{\alpha\beta}{}^\gamma c_\gamma \partial_{c_\alpha} \partial_{c_\beta}, $$

$$\Delta_{-3} = \frac{1}{3!}(-1)^{p(c_\beta)+1} f_{\gamma\beta\alpha} \partial_{c_\alpha} \partial_{c_\beta} \partial_{c_\gamma}, \tag{2.7}$$

$$\tilde{\Delta}_{-3} = \frac{1}{3!}(-1)^{p(c_\beta)+1} f_{\alpha\beta\gamma} \partial_{c_\alpha} \partial_{c_\beta} \partial_{c_\gamma}. \tag{2.8}$$

In the relations (2.4) and (2.5) c^α are coordinates in the adjoint representation of G which for an arbitrary semi-simple Lie superalgebra connected with co-adjoint representation with the help of the metric $g^{\beta\alpha}$

$$c^\alpha = c_\beta g^{\beta\alpha}$$

that is inverse

$$g_{\alpha\beta} g^{\beta\gamma} = \delta^\gamma_\alpha$$

to the Cartan-Killing metric tensor

$$g_{\alpha\beta} = (-1)^{p(z_\lambda)} f_{\alpha\gamma}{}^\lambda f_{\beta\lambda}{}^\gamma.$$

The tensor with the low indices

$$f_{\alpha\beta\gamma} = f_{\alpha\beta}{}^\lambda g_{\lambda\gamma}$$

in the relations (2.7) and (2.8), due to the equations (2.2) and (2.3), has the following symmetry properties:

$$f_{\alpha\beta\gamma} = -(-1)^{p(z_\alpha)p(z_\beta)} f_{\beta\alpha\gamma} = -(-1)^{p(z_\beta)p(z_\gamma)} f_{\alpha\gamma\beta}.$$

The operator Δ_{+1} (2.4) is proportional to the second terms in the BRST-like nilpotent charges

$$c^\alpha G_\alpha - \Delta_{+1},$$

$$(-1)^{p(c^\alpha)+1} c^\alpha G_\alpha - \Delta_{+1}.$$

The operator Δ_{-1} (2.6) related to the divergence of the vector field $\{c_\alpha, A\}_1$

$$\Delta_{-1} A = \frac{1}{2} (-1)^{p(c_\alpha)} \partial_{c_\alpha} \{c_\alpha, A\}_1$$

is proportional to the true Batalin-Vilkovisky Δ-operator [2, 3] for the linear odd Poisson bracket (2.1) and to the second terms in the following nilpotent charges:

$$G_\alpha \partial_{c_\alpha} - \Delta_{-1},$$

$$(-1)^{p(c_\alpha)+1} G_\alpha \partial_{c_\alpha} - \Delta_{-1}.$$

The quantities c^α and ∂_{c^α} represent the operators for the ghosts and ghost momenta, respectively. The operator Δ_{-1} (2.6) determines the linear odd Poisson bracket (2.1) as a deviation of the Leibniz rule under the usual multiplication

$$\Delta_{-1}(A \cdot B) = (\Delta_{-1} A) \cdot B + (-1)^{p(A)} A \cdot \Delta_{-1} B + (-1)^{p(A)} \{A, B\}_1 .$$

and simultaneously satisfies the Leibniz rule with respect to the linear odd Poisson bracket composition

$$\Delta_{-1}(\{A, B\}_1) = \{\Delta_{-1} A, B\}_1 + (-1)^{p(A)+1} \{A, \Delta_{-1} B\}_1.$$

In the present paper we show that the operators Δ_{-3} (2.7) and $\tilde{\Delta}_{-3}$ (2.8) are related with the Grassmann-odd Nambu-like brackets [5] on the supermanifolds. These brackets correspond to the arbitrary Lie superalgebra.

3. Nambu-like odd brackets

By applying the operators Δ_{-3} (2.7) and $\tilde{\Delta}_{-3}$ (2.8) to the usual product of two quantities A and B, we obtain

$$\Delta_{-3}(A \cdot B) = (\Delta_{-3}A) \cdot B + (-1)^{p(A)}A \cdot \Delta_{-3}B + (-1)^{p(A)}\Delta(A,B),$$

$$\tilde{\Delta}_{-3}(A \cdot B) = (\tilde{\Delta}_{-3}A) \cdot B + (-1)^{p(A)}A \cdot \tilde{\Delta}_{-3}B + (-1)^{p(A)}\tilde{\Delta}(A,B),$$

where the values $\Delta(A,B)$ and $\tilde{\Delta}(A,B)$ are

$$\Delta(A,B) = \frac{1}{2}f_{\gamma\beta\alpha}(-1)^{p(c_\beta)+1}\Delta\left(\partial_{\theta_\alpha}A, \partial_{\theta_\gamma}C\right)\left[(-1)^{p(A)[p(c_\gamma)+1]}\left(\partial_{c_\alpha}\partial_{c_\beta}A\right)\partial_{c_\gamma}B\right.$$
$$\left. + (-1)^{p(A)p(c_\alpha)}\left(\partial_{c_\alpha}A\right)\partial_{c_\beta}\partial_{c_\gamma}B\Delta\left(\partial_{\theta_\alpha}A, \partial_{\theta_\gamma}C\right)\right], \tag{3.1}$$

$$\tilde{\Delta}(A,B) = \frac{1}{2}f_{\alpha\beta\gamma}(-1)^{p(c_\beta)+1}\Delta\left(\partial_{\theta_\alpha}A, \partial_{\theta_\gamma}C\right)\left[(-1)^{p(A)[p(c_\gamma)+1]}\left(\partial_{c_\alpha}\partial_{c_\beta}A\right)\partial_{c_\gamma}B\right.$$
$$\left. + (-1)^{p(A)p(c_\alpha)}\left(\partial_{c_\alpha}A\right)\partial_{c_\beta}\partial_{c_\gamma}B\Delta\left(\partial_{\theta_\alpha}A, \partial_{\theta_\gamma}C\right)\right]. \tag{3.2}$$

By acting with the operators Δ_{-3} (2.7) and $\tilde{\Delta}_{-3}$ (2.8) on the usual product of three quantities A, B and C, we come to the following relations:

$$\Delta_{-3}(A \cdot B \cdot C) = (\Delta_{-3}A) \cdot B \cdot C + (-1)^{p(A)}A \cdot (\Delta_{-3}B) \cdot C + (-1)^{p(A)+p(B)}A \cdot B \cdot \Delta_{-3}C$$
$$+ (-1)^{p(A)}\Delta(A,B)C + (-1)^{p(A)p(B)+p(A)+p(B)}B\Delta(A,C)$$
$$+ (-1)^{p(A)+p(B)}A\Delta(B,C) + (-1)^{p(B)}\{A,B,C\}_1, \tag{3.3}$$

$$\tilde{\Delta}_{-3}(A \cdot B \cdot C) = \left(\tilde{\Delta}_{-3}A\right) \cdot B \cdot C + (-1)^{p(A)}A \cdot \left(\tilde{\Delta}_{-3}B\right) \cdot C + (-1)^{p(A)+p(B)}A \cdot B \cdot \tilde{\Delta}_{-3}C$$
$$+ (-1)^{p(A)}\tilde{\Delta}(A,B)C + (-1)^{p(A)p(B)+p(A)+p(B)}B\tilde{\Delta}(A,C)$$
$$+ (-1)^{p(A)+p(B)}A\tilde{\Delta}(B,C) + (-1)^{p(B)}\{A,B,C\}_{\tilde{1}}, \tag{3.4}$$

where the last terms in the right hand sides of the (3.3) and (3.4) are the Grassmann-odd Nambu-like brackets

$$\{A,B,C\}_1 = (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]+p(c_\beta)+1}f_{\gamma\beta\alpha}\partial_{c_\alpha}A\partial_{c_\beta}B\partial_{c_\gamma}C \tag{3.5}$$

$$\{A,B,C\}_{\tilde{1}} = (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]+p(c_\beta)+1}f_{\alpha\beta\gamma}\partial_{c_\alpha}A\partial_{c_\beta}B\partial_{c_\gamma}C \tag{3.6}$$

on the supermanifolds with coordinates c_α.

The divergences of the Nambu-like odd brackets (3.5) and (3.6) are related with the values $\Delta(A,B)$ (3.1) and $\tilde{\Delta}(A,B)$ (3.2) (see also [11])

$$\Delta(A,B) = \frac{1}{2}\partial_{c_\alpha}\{c_\alpha, A, B\}_1,$$

$$\tilde{\Delta}(A,B) = \frac{1}{2}\partial_{c_\alpha}\{c_\alpha, A, B\}_{\tilde{1}}.$$

The contraction of the Grassmann-odd Nambu-like bracket (3.5) with the variable c^α gives the linear odd Poisson bracket (2.1)

$$\{A, B\}_1 = -c^\alpha \{c_\alpha, A, B\}_1,$$

whereas the contraction of the odd Nambu-like bracket (3.6) with c^α gives the expression

$$c^\alpha \{c_\alpha, A, B\}_{\tilde{1}} = (-1)^{p(c_\alpha)p(c_\beta)+1} A \overleftarrow{\partial}_{c_\alpha} f_{\alpha\beta}{}^\gamma c_\gamma \overrightarrow{\partial}_{c_\beta} B,$$

which is not a Poisson bracket, since it does not obey the Jacobi identity. Note also the following relations between the operators Δ_{-3} (2.7), $\tilde{\Delta}_{-3}$ (2.8) and odd Nambu-like brackets (3.6), (3.5):

$$\Delta_{-3}A = \frac{1}{3!}(-1)^{p(c_\alpha)}\partial_{c_\alpha}\partial_{c_\beta}\{c_\alpha, c_\beta, A\}_{\tilde{1}},$$

$$\tilde{\Delta}_{-3}A = \frac{1}{3!}(-1)^{p(c_\alpha)}\partial_{c_\alpha}\partial_{c_\beta}\{c_\alpha, c_\beta, A\}_1.$$

For the operator Δ_{-3} (2.7) and $\tilde{\Delta}_{-3}$ (2.8) there exists the following "Leibniz rules" with respect to the odd Nambu-like bracket compositions:

$$
\begin{aligned}
\Delta_{-3}(\{A, B, C\}_1) = & - \{\Delta_{-3}A, B, C\}_1 + (-1)^{p(A)}\{A, \Delta_{-3}B, C\}_1 \\
& - (-1)^{p(A)+p(B)}\{A, B, \Delta_{-3}C\}_1 \\
& + (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]} f_{\gamma\beta\alpha} \\
& \times \Big[(-1)^{p(A)+p(c_\gamma)}\Delta\left(\partial_{c_\alpha}A, \partial_{c_\beta}B\right)\partial_{c_\gamma}C \\
& + (-1)^{p(A)+p(B)+p(c_\alpha)+1}\partial_{c_\alpha}A\Delta\left(\partial_{c_\beta}B, \partial_{c_\gamma}C\right) \\
& + (-1)^{p(A)p(B)+p(c_\beta)+p(A)[p(c_\beta)+1]+p(B)[p(c_\alpha)+1]+[p(c_\alpha)+1][p(c_\beta)+1]} \\
& \times \partial_{c_\beta}B\Delta\left(\partial_{c_\alpha}A, \partial_{c_\gamma}C\right)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\Delta}_{-3}(\{A, B, C\}_{\tilde{1}}) = & - \{\tilde{\Delta}_{-3}A, B, C\}_{\tilde{1}} + (-1)^{p(A)}\{A, \tilde{\Delta}_{-3}B, C\}_{\tilde{1}} \\
& - (-1)^{p(A)+p(B)}\{A, B, \tilde{\Delta}_{-3}C\}_{\tilde{1}} \\
& + (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]} f_{\alpha\beta\gamma} \\
& \times \Big[(-1)^{p(A)+p(c_\gamma)}\tilde{\Delta}\left(\partial_{c_\alpha}A, \partial_{c_\beta}B\right)\partial_{c_\gamma}C \\
& + (-1)^{p(A)+p(B)+p(c_\alpha)+1}\partial_{c_\alpha}A\tilde{\Delta}\left(\partial_{c_\beta}B, \partial_{c_\gamma}C\right) \\
& + (-1)^{p(A)p(B)+p(c_\beta)+p(A)[p(c_\beta)+1]+p(B)[p(c_\alpha)+1]+[p(c_\alpha)+1][p(c_\beta)+1]} \\
& \times \partial_{c_\beta}B\tilde{\Delta}\left(\partial_{c_\alpha}A, \partial_{c_\gamma}C\right)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{-3}(\{A, B, C\}_{\tilde{1}}) = & - \{\Delta_{-3}A, B, C\}_{\tilde{1}} + (-1)^{p(A)}\{A, \Delta_{-3}B, C\}_{\tilde{1}} \\
& - (-1)^{p(A)+p(B)}\{A, B, \Delta_{-3}C\}_{\tilde{1}} \\
& + (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]} f_{\alpha\beta\gamma} \\
& \times \Big[(-1)^{p(A)+p(c_\gamma)}\Delta\left(\partial_{c_\alpha}A, \partial_{c_\beta}B\right)\partial_{c_\gamma}C \\
& + (-1)^{p(A)+p(B)+p(c_\alpha)+1}\partial_{c_\alpha}A\Delta\left(\partial_{c_\beta}B, \partial_{c_\gamma}C\right)
\end{aligned}
$$

$$+ (-1)^{p(A)p(B)+p(c_\beta)+p(A)[p(c_\beta)+1]+p(B)[p(c_\alpha)+1]+[p(c_\alpha)+1][p(c_\beta)+1]}$$
$$\times \partial_{c_\beta} B \Delta \left(\partial_{c_\alpha} A, \partial_{c_\gamma} C \right) \Big],$$

$$\tilde{\Delta}_{-3}(\{A,B,C\}_1) = - \{\tilde{\Delta}_{-3}A, B, C\}_1 + (-1)^{p(A)}\{A, \tilde{\Delta}_{-3}B, C\}_1$$
$$- (-1)^{p(A)+p(B)}\{A, B, \tilde{\Delta}_{-3}C\}_1$$
$$+ (-1)^{p(A)[p(c_\alpha)+1]+p(B)[p(c_\gamma)+1]} f_{\gamma\beta\alpha}$$
$$\times \Big[(-1)^{p(A)+p(c_\gamma)} \tilde{\Delta} \left(\partial_{c_\alpha} A, \partial_{c_\beta} B \right) \partial_{c_\gamma} C$$
$$+ (-1)^{p(A)+p(B)+p(c_\alpha)+1} \partial_{c_\alpha} A \tilde{\Delta} \left(\partial_{c_\beta} B, \partial_{c_\gamma} C \right)$$
$$+ (-1)^{p(A)p(B)+p(c_\beta)+p(A)[p(c_\beta)+1]+p(B)[p(c_\alpha)+1]+[p(c_\alpha)+1][p(c_\beta)+1]}$$
$$\times \partial_{c_\beta} B \tilde{\Delta} \left(\partial_{c_\alpha} A, \partial_{c_\gamma} C \right) \Big].$$

The Grassmann parity

$$p(\{A,B,C\}) = p(A) + p(B) + p(C) + 1 \pmod 2,$$

symmetry properties

$$\{A,B,C\} = -(-1)^{[p(A)+1][p(B)+1]}\{B,A,C\} = -(-1)^{[p(B)+1][p(C)+1]}\{A,C,B\}$$

and Jacobi type identity

$$\{\{A,B,C\},D,E\} + (-1)^{[p(A)+1][p(B)+p(C)+p(D)+p(E)]}\{\{B,C,D\},E,A\}$$
$$+ (-1)^{[p(A)+p(B)][p(C)+p(D)+p(E)+1]}\{\{C,D,E\},A,B\}$$
$$+ (-1)^{[p(D)+p(E)][p(A)+p(B)+p(C)+1]}\{\{D,E,A\},B,C\}$$
$$+ (-1)^{[p(E)+1][p(A)+p(B)+p(C)+p(D)]}\{\{E,A,B\},C,D\}$$
$$+ (-1)^{[p(D)+p(E)][p(A)+p(B)+p(C)+1]+p(B)[p(A)+1]+p(A)}$$
$$\times \{\{D,E,B\},A,C\}$$
$$+ (-1)^{[p(D)+1][p(A)+p(B)+p(C)]+p(D)}\{\{D,A,B\},C,E\}$$
$$+ (-1)^{[p(A)+1][p(B)+p(C)+p(D)+p(E)]+[p(D)+1]p(E)+p(D)}$$
$$\times \{\{B,C,E\},D,A\}$$
$$+ (-1)^{[p(B)+1][p(C)+p(D)+p(E)]+p(B)}\{\{A,C,D\},E,B\}$$
$$+ (-1)^{[p(B)+1][p(C)+p(D)+p(E)+1]+[p(D)+1][p(E)+1]}$$
$$\times \{\{A,C,E\},D,B\} = 0 \tag{3.7}$$

follow from the expressions (3.5) and (3.6) for the odd Nambu-like brackets, where either

$$\{A,B,C\} \equiv \{A,B,C\}_1$$

or

$$\{A,B,C\} \equiv \{A,B,C\}_{\tilde{1}}.$$

Note that the structure of (3.7) is different from the structure of the "fundamental identity" [12], which has four terms, and, containing ten terms, is similar to the structure of the "generalized Jacobi identity" [13, 14, 15] and to the "(J2)-structure" [16], which is intimately related to the homotopy algebras [17] and SH-algebras [18].

4. Conclusion

Thus, by using the Grassmann-odd nilpotent differential operators Δ_{-3} (2.7) and $\tilde{\Delta}_{-3}$ (2.8) of the third order with respect to the derivatives, we constructed two different Grassmann-odd Nambu-like brackets (3.5) and (3.6), respectively, which correspond to the arbitrary Lie superalgebra and are realized on the supermanifolds. The main properties of these brackets are also given.

It would be interesting to consider the dynamics based on these brackets. The work in this direction is in progress.

Acknowledgments

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